

## Specimen Paper WMA13/01

Question	Scheme	Marks
<b>1(a)</b>	$(3, -1)$	B1B1
		<b>(2)</b>
<b>(b)</b>	$\left(-\frac{7}{2}, 5\right)$	B1B1
		<b>(2)</b>
		<b>(4 marks)</b>

**Notes:**

**(a)**

**B1** Correct  $x$  coordinate

**B1** Correct  $y$  coordinate

**(b)**

**B1** Correct  $x$  coordinate

**B1** Correct  $y$  coordinate

## Specimen Paper WMA13/01

Question	Scheme	Marks
<b>2(a)</b>	$\cos(A + A) = \cos^2 A - \sin^2 A = (1 - \sin^2 A) - \sin^2 A$ $\cos 2A = 1 - 2\sin^2 A \quad *$	M1 A1*
		<b>(2)</b>
<b>(b)</b>	$\int_0^{\frac{\pi}{4}} 3\sin^2 2x \, dx = 3 \int_0^{\frac{\pi}{4}} \left( \frac{1}{2} - \frac{\cos 4x}{2} \right) dx$	M1
	$\left[ \frac{3}{2}x - \frac{3}{8}\sin 4x \right]_0^{\frac{\pi}{4}} = \left( \frac{3}{2} \times \frac{\pi}{4} - 0 \right) - 0$	M1
	$= \frac{3\pi}{8}$	A1
		<b>(3)</b>
		<b>(5 marks)</b>

**Notes:**

**(a)**

**M1** Attempts the compound angle formulae and applies  $\cos^2 A = 1 - \sin^2 A$

**A1\*** Achieves the given answer with no errors or missing brackets.

**(b)**

**M1** Attempts to replace  $\sin^2 2x$  with  $\left( \frac{1}{2} - \frac{1}{2} \cos 4x \right)$ . As a minimum look for an expression of the form  $\int a \pm b \cos 4x \, dx$ .

**M1** Integrates  $\dots \cos 4x \rightarrow \dots \sin 4x$  **and** substitutes in the limits of  $\frac{\pi}{4}$  and 0 into their changed function (subtracts either way round)

**A1**  $\frac{3\pi}{8}$  cao

**Note an answer with no working in (b) is 0 marks**

## Specimen Paper WMA13/01

Question	Scheme	Marks
<b>3(a)</b>	$a + 60e^{-0.05 \times 0} = 2(100 + 80e^{0.05 \times 0})$	M1
	$a = 200 + 160 - 60 = 300$	A1
		<b>(2)</b>
<b>(b)</b>	$"300" + 60e^{-0.05T} = 100 + 80e^{0.05T} \Rightarrow \dots e^{0.05T} \pm \dots e^{-0.05T} \pm \dots = 0$	M1
	$4e^{0.1T} - 10e^{0.05T} - 3 = 0 \Rightarrow e^{0.05T} = \frac{10 + \sqrt{148}}{8}$ oe	M1
	$\Rightarrow T = \frac{\ln(\dots)}{0.05}$	dM1
	$T = 20.3819\dots = 20.4$	A1
		<b>(4)</b>
<b>(6 marks)</b>		

### Notes:

**(a)**

**M1** Sets the number of guinea pigs equal to  $2 \times$  number of rabbits and sets  $t = 0$

**A1** 300

**(b)**

**M1** Sets their  $"300" + 60e^{-0.05T} = 100 + 80e^{0.05T}$  and rearranges to produce a simplified equation of the form  $\dots e^{0.05T} \pm \dots e^{-0.05T} \pm \dots = 0$

**M1** Sets up and solves their 3TQ in  $e^{0.05T}$

**dM1** Depends on previous M mark. Solves their  $e^{0.05T} = C$ ,  $C > 0$  using lns to find value for  $T$

**A1** 20.4 only

## Specimen Paper WMA13/01

Question	Scheme	Marks
<b>4</b>	<p>Possible solutions include:</p> <p>Solution from <math>R \sin(2\theta + \alpha)</math> or <math>R \cos(2\theta - \alpha)</math></p> <p>Eg Attempt at <math>3 \sin 2\theta + 5 \cos 2\theta = R \sin(2\theta + \alpha)</math></p> $R = \sqrt{3^2 + 5^2} \quad (= \sqrt{34})$ $\alpha = \tan^{-1}\left(\frac{5}{3}\right) \Rightarrow \alpha = \dots$ $\alpha = \text{awrt } 1.03$ $" \sqrt{34} " \sin(2\theta \pm "1.03") = 4 \Rightarrow \sin(2\theta \pm "1.03") = \frac{4}{" \sqrt{34} "}$ $2\theta \pm "1.03" = \sin^{-1}\left(\frac{4}{" \sqrt{34} "}\right) \Rightarrow \theta_1 = \frac{(2n+1)\pi - \sin^{-1}\left(\frac{4}{" \sqrt{34} "}\right) \mp "1.03"}{2}$ $\text{or } \theta_2 = \frac{\sin^{-1}\left(\frac{4}{" \sqrt{34} "}\right) + 2n\pi \mp "1.03"}{2}$ $\theta = \text{awrt } 0.68, 3.0, 3.8, 6.1$	<p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>dM1</p> <p>A1A1</p>
<b>Alt1</b>	<p>Solution using double angle formulae, e.g</p> $3 \sin 2\theta + 5 \cos 2\theta = 4$ $6 \sin \theta \cos \theta + 5(2 \cos^2 \theta - 1) = 4$ $10 \cos^2 \theta + 6 \sin \theta \cos \theta = 9$ $10 + 6 \tan \theta = 9 \sec^2 \theta$ $10 + 6 \tan \theta = 9(1 + \tan^2 \theta)$	<p>M1</p> <p>M1A1</p> <p>M1</p>

## Specimen Paper WMA13/01

	$9 \tan^2 \theta - 6 \tan \theta - 1 = 0$ $\tan \theta = \frac{6 \pm \sqrt{72}}{18}$ $\theta = \text{awrt } 0.68, 3.0, 3.8, 6.1$	dM1  A1A1
<b>Alt2</b>	Solution using trigonometric identities, e.g  $3 \sin 2\theta + 5 \cos 2\theta = 4$ $3 \tan 2\theta + 5 = 4 \sec 2\theta$ $9 \tan^2 2\theta + 30 \tan 2\theta + 25 = 16 \sec^2 2\theta$ $9 \tan^2 2\theta + 30 \tan 2\theta + 25 = 16(1 + \tan^2 2\theta)$ $7 \tan^2 2\theta - 30 \tan 2\theta - 9 = 0$ $\tan 2\theta = \frac{30 \pm \sqrt{1152}}{14} = -0.2815, 4.567$ $\theta = \frac{\arctan "-0.2815"}{2} \text{ or } \theta = \frac{\arctan "4.567"}{2}$ $\theta = \text{awrt } 0.68, 3.0, 3.8, 6.1$	M1  M1  A1  M1  dM1  A1A1
<b>(7 marks)</b>		

### Notes:

There are many ways to solving this question. Examples of three types of solution are given in the main scheme with the notes below. Other ways can be marked similarly.

Attempt using  $R \sin(2\theta \pm \alpha)$

**M1** Attempts to find  $R$  (may be given on sight of  $\sqrt{3^2+5^2}$  or  $\sqrt{34}$ )

**M1** For sight of  $\tan \alpha = \pm \frac{5}{3}$ ,  $\tan \alpha = \pm \frac{3}{5}$ . Condone  $\sin \alpha = 5$ ,  $\cos \alpha = 3 \Rightarrow \tan \alpha = \frac{5}{3}$ .

If  $R$  is found first, accept  $\sin \alpha = \pm \frac{5}{R}$ ,  $\cos \alpha = \pm \frac{3}{R}$

**A1**  $\alpha = \text{awrt } 1.03$ . (The degree equivalent  $59.0^\circ$  is A0)

**M1** Sets their " $\sqrt{34}$ "  $\sin(2\theta \pm "1.03") = 4$  and proceeds to  $\sin(2\theta \pm "1.03") = \frac{4}{\sqrt{34}}$

## Specimen Paper WMA13/01

- M1** A correct method to find one of the solutions. As a minimum they should find  $2n\pi + \sin^{-1}\left(\frac{4}{\sqrt{34}}\right)$  where  $n = 1$  or  $2$  and proceeds correctly to find  $\theta = \dots$  (awrt 3.0 or awrt 6.1 is sufficient evidence of this). Alternatively they find  $n\pi - \sin^{-1}\left(\frac{4}{\sqrt{34}}\right)$  where  $n = 1$  or  $2$  and proceeds correctly to find  $\theta = \dots$  (awrt 0.68 or awrt 3.8 is sufficient evidence of this)
- A1** Any two of  $\theta =$  awrt 0.68, 3.0, 3.8, 6.1
- A1**  $\theta =$  awrt 0.68, 3.0, 3.8, 6.1 only
- 

- Alt1** Attempt using double angle formulae
- M1** Uses both  $\sin 2\theta = 2 \sin \theta \cos \theta$  and  $\cos 2\theta = 2 \cos^2 \theta - 1$
- M1** Divides by  $\cos^2 \theta$  to form an equation in  $\tan \theta$  and  $\sec \theta$
- A1** Correct equation
- M1** Uses  $1 + \tan^2 \theta = \sec^2 \theta$  to form a quadratic equation in  $\sec \theta$
- M1** A correct method to find one of the values of  $\theta$
- A1** Any two of  $\theta =$  awrt 0.68, 3.0, 3.8, 6.1
- A1**  $\theta =$  awrt 0.68, 3.0, 3.8, 6.1 only
- 

- Alt2** Attempt using trigonometric identities
- M1** Divides by  $\cos 2\theta$  to form an equation in  $\tan 2\theta$  and  $\sec 2\theta$
- M1** Attempts to square and uses the identity  $1 + \tan^2 2\theta = \sec^2 2\theta$
- A1** Correct equation
- M1** Attempts to solve the quadratic equation to find at least one value for  $\tan 2\theta$
- M1** A correct method to find one of the values of  $\theta$
- A1** Any two of  $\theta =$  awrt 0.68, 3.0, 3.8, 6.1
- A1**  $\theta =$  awrt 0.68, 3.0, 3.8, 6.1 only

## Specimen Paper WMA13/01

Question	Scheme	Marks
<b>5(a)</b>	$\frac{7}{x+3} - \frac{5x+22}{(x+3)(x+4)} = \frac{7(x+4) - 5x + 22}{(x+3)(x+4)}$	M1
	$= \frac{7x+28-5x+22}{(x+3)(x+4)} = \frac{2x+6}{(x+3)(x+4)}$	M1
	$= \frac{2(x+3)}{(x+3)(x+4)} = \frac{2}{x+4}$	A1
<b>(3)</b>		
<b>(b)</b>	$y = \frac{2}{x+4} \Rightarrow x = \dots$	M1
	$\Rightarrow f^{-1}(x) = \frac{2}{x} - 4$	A1
	$(x \in \square, ) 0 < x < 2$	B1
<b>(3)</b>		
<b>(c)</b>	$\{ff(x) = \} \frac{2}{\frac{2}{x+4} + 4} = \frac{2}{5}$	B1
	$10(x+4) = 2(2+4x+16) \Rightarrow x = \dots$	M1
	$x = -2$	A1
<b>(3)</b>		
<b>(9 marks)</b>		

**Notes:**

**(a)**

**M1** Writes both fractions with the same common denominator or writes as a single fraction

**M1** Simplifies the numerator and denominator to a form of  $\frac{\text{linear}}{\text{quadratic}}$

## Specimen Paper WMA13/01

**A1** Cancels to achieve the required answer cao

**(b)**

**M1** Correct method to find the inverse.

**A1**  $f^{-1}(x) = \frac{2}{x} - 4$

**B1**  $(x \in \square, ) 0 < x < 2$

**(c)**

**B1** Sight of the correct starting equation. This may be implied by later working.

Note an alternative method via  $f(x) = f^{-1}\left(\frac{2}{5}\right) \Rightarrow \frac{2}{x+4} = 1$

**M1** A correct method to find  $x$ .

**A1**  $x = -2$



## Specimen Paper WMA13/01

Question	Scheme	Marks
<b>6(a)</b>	<p>At <math>P</math> <math>x &gt; \pi \Rightarrow y = -\sin x + 1</math></p> $\frac{dy}{dx} = -\cos x = -\frac{1}{2} \Rightarrow x = \dots$ $a = \frac{5\pi}{3}$ <p><math>y</math> coordinate of <math>P = \left  \sin \frac{5\pi}{3} \right  + 1 = \dots</math></p> $b = \frac{\sqrt{3}}{2} + 1$	<p>M1</p> <p>A1</p> <p>dM1</p> <p>A1</p>
	<b>(4)</b>	
<b>(b)</b>	$m = \frac{\left  \frac{\sqrt{3}}{2} + 1 \right ^{-1}}{\left  \frac{5\pi}{3} \right  - 0} \text{ or } m = \frac{\left  \frac{\sqrt{3}}{2} + 1 \right ^{-1}}{\left  \frac{5\pi}{3} \right  - \pi}$ $m = \frac{\left  \frac{\sqrt{3}}{2} + 1 \right ^{-1}}{\left  \frac{5\pi}{3} \right  - 0} \text{ and } m = \frac{\left  \frac{\sqrt{3}}{2} + 1 \right ^{-1}}{\left  \frac{5\pi}{3} \right  - \pi}$ $\frac{3\sqrt{3}}{10\pi} < m < \frac{3\sqrt{3}}{4\pi}$	<p>M1</p> <p>M1</p> <p>A1</p>
	<b>(3)</b>	
<b>(7 marks)</b>		

**Notes:**

**(a)**

**M1** Differentiates  $\pm \sin x$  to achieve  $\pm \cos x = \pm \frac{1}{2}$  and proceeds to achieve an angle for  $x$ .

May come from symmetry or periodicity of  $\sin x$

**A1**  $a = \frac{5\pi}{3}$

**dM1** Substitutes their value of  $a$  for  $x$  to find  $b$ .

## Specimen Paper WMA13/01

**A1**  $b = \frac{\sqrt{3}}{2} + 1$

**(b)**

**M1** Attempts to find the gradient of the line between their  $P$  and either  $(0,1)$  or  $(\pi,1)$

**M1** Attempts to find the gradient of the line between their  $P$  and both  $(0,1)$  and  $(\pi,1)$

**A1**  $\frac{3\sqrt{3}}{10\pi} < m < \frac{3\sqrt{3}}{4\pi}$  or  $\frac{3\sqrt{3}}{10\pi} < m$  AND  $m < \frac{3\sqrt{3}}{4\pi}$  or  $\left(\frac{3\sqrt{3}}{10\pi}, \frac{3\sqrt{3}}{4\pi}\right)$  or exact equivalent.

DO NOT accept  $\frac{3\sqrt{3}}{10\pi} < m$  OR  $m < \frac{3\sqrt{3}}{4\pi}$

## Specimen Paper WMA13/01

Question	Scheme	Marks
<b>7(a)</b>	$\frac{dy}{dx} = \dots e^{2\sqrt{3}x} \cos 2x \pm \dots e^{2\sqrt{3}x} \sin 2x$	M1
	$\frac{dy}{dx} = 2\sqrt{3}e^{2\sqrt{3}x} \cos 2x - 2e^{2\sqrt{3}x} \sin 2x$	A1
		<b>(2)</b>
<b>(b)</b>	$"2\sqrt{3}e^{2\sqrt{3}x} \cos 2x - 2e^{2\sqrt{3}x} \sin 2x" = 0 \Rightarrow$ $e^{2\sqrt{3}x} (\dots \cos 2x - \dots \sin 2x) = 0$	M1
	$\dots \cos 2x - \dots \sin 2x = 0 \Rightarrow \tan 2x = "\sqrt{3}"$	M1
	$2x = \tan^{-1}("\sqrt{3}") \Rightarrow x = \dots$	M1
	$x = \frac{\pi}{6}, \frac{2\pi}{3}$	A1
	$y = e^{\left(2\sqrt{3}x \times \frac{\pi}{6}\right)} \cos\left(2 \times \frac{\pi}{6}\right) = \dots$	M1
	$\left(\frac{\pi}{6}, \frac{1}{2}e^{\frac{\sqrt{3}\pi}{3}}\right)$ and $\left(\frac{2\pi}{3}, -\frac{1}{2}e^{\frac{4\sqrt{3}\pi}{3}}\right)$	A1
		<b>(6)</b>
		<b>(8 marks)</b>

### Notes:

**(a)**

**M1** Attempts to use the product rule to achieve an expression of the form

$$\frac{dy}{dx} = \dots e^{2\sqrt{3}x} \cos 2x \pm \dots e^{2\sqrt{3}x} \sin 2x$$

**A1** Correct unsimplified expression.

**(b)**

**M1** Sets their  $\frac{dy}{dx} = 0$  and attempts to factorise by taking out the exponential term.

**M1** Attempts to solve their factorised expression/rearranges their trigonometric solution to  $\tan 2x = \dots$

**M1** Attempts to find one angle from their trigonometric equation.

## Specimen Paper WMA13/01

- A1** Both  $x$  coordinates correct.
- M1** Attempts to find the corresponding  $y$  coordinate for one of their  $x$  coordinates.
- A1** Both coordinates correct (accept any exact equivalent forms).

## Specimen Paper WMA13/01

Question	Scheme	Marks
<b>8(a)</b>	$P = ab^{-t}$ $\log_{10} P = \log_{10} ab^{-t} \Rightarrow \log_{10} P = \log_{10} a + \log_{10} b^{-t}$ $\Rightarrow \log_{10} P = \log_{10} a - t \log_{10} b^*$	B1*
		<b>(1)</b>
<b>(b)</b>	$\log_{10} a = 1.6 \Rightarrow a = \text{awrt } 39.81$ <p>Using (10,1.4), <math>t = 10</math>, <math>\log_{10} P = 1.4</math></p> $\log_{10} P = \log_{10} a - 10 \log_{10} b \Rightarrow 1.4 = 1.6 - 10 \log_{10} b$ $1.4 = 1.6 - 10 \log_{10} b \Rightarrow \log_{10} b = "0.02" \Rightarrow b = 10^{0.02}$ $b = \text{awrt } 1.047$	B1  M1  M1  A1
		<b>(4)</b>
<b>(c)</b>	$\frac{dP}{dt} = -a \ln b \times b^{-t}$ $\frac{dP}{dt} = -"39.81" \times \ln("1.047") \times "1.047"^{-8}$ $\frac{dP}{dt} = -1.26836... \Rightarrow \text{decrease of awrt } 1270 \text{ people per year}$	M1  M1  A1
		<b>(3)</b>
<b>(8 marks)</b>		
<b>Alt1(b)</b>	$\log_{10} a = 1.6 \Rightarrow a = \text{awrt } 39.81$ $-\log_{10} b = \frac{1.4 - 1.6}{10 - 0} = ...$ $\log_{10} b = "0.02" \Rightarrow b = 10^{0.02}$ $b = \text{awrt } 1.047$	B1  M1  M1  A1

**Notes:**

**(a)**

**B1\*** Must see **taking lns of both sides, using the addition rule** to write as two separate logs and **using the power rule** to achieve the given answer with no errors.

## Specimen Paper WMA13/01

(b)

**B1** awrt 39.81

**M1** Uses (10,1.4) by substituting into the equation of the line. Alternatively attempts to calculate the gradient of the line using (0,1.6) and (10,1.4)

**M1** Rearranges to make  $\log_{10} b$  the subject and carries out correct log work to find a value for  $b$ . Alternatively sets their gradient equal to  $\log_{10} b$  and proceeds correctly to find  $b$ .

**A1** awrt 1.047

(c)

**M1** Differentiates to find  $\frac{dP}{dt}$  in the form  $\dots \ln b \times b^{-t}$  with their values from (b)

**M1** Substitutes  $t = 8$  into their  $\frac{dP}{dt}$  (it must be a changed expression i.e it cannot be the original equation).

**A1** decrease of awrt 1270 people per year. Allow  $-1270$  people per year . Must include units.

## Specimen Paper WMA13/01

Question	Scheme	Marks
<b>9(a)</b>	$x = \frac{(\sin y - \cos y)(\sin y + \cos y)}{(\cos y + \sin y)(\cos y + \sin y)} \Rightarrow x = \frac{\sin^2 y - \cos^2 y}{\cos^2 y + 2 \sin y \cos y + \sin^2 y}$ $\Rightarrow x = \frac{-\cos 2y}{1 + \sin 2y} \quad *$	M1  M1A1*
		<b>(3)</b>
<b>(b)</b>	<p>Quotient rule:</p> $\frac{dx}{dy} = \frac{(1 + \sin 2y) \times 2 \sin 2y - (-\cos 2y) \times 2 \cos 2y}{(1 + \sin 2y)^2}$ $\frac{dx}{dy} = \frac{2 \sin 2y + 2 \sin^2 2y + 2 \cos^2 y}{(1 + \sin 2y)^2} = \frac{2 \sin 2y + 2}{(1 + \sin 2y)^2}$ $\Rightarrow \frac{dx}{dy} = \frac{2}{1 + \sin 2y} \quad *$	M1   A1*
		<b>(2)</b>
<b>(c)</b>	$\frac{1 + \sin 2y}{2} = \frac{1}{4} \Rightarrow \sin 2y = \dots$ $\sin 2y = \frac{-1}{2} \Rightarrow y = \frac{1}{2} \sin^{-1} \left( \frac{-1}{2} \right)$ $y = -\frac{\pi}{12}$ $x = \frac{-\cos \left( -\frac{\pi}{6} \right)}{1 + \sin \left( -\frac{\pi}{6} \right)} \Rightarrow x = \dots$ $x = -\sqrt{3}$	M1  M1  A1  M1  A1
		<b>(5)</b>
<b>(10 marks)</b>		
<b>Alt1(b)</b>	<p>Product rule</p> $x = -\cos 2y(1 + \sin 2y)^{-1}$	





## Specimen Paper WMA13/01

In the alternative method they attempt the product rule. If the formula is not quoted then look for an expression of the form

$$\frac{dx}{dy} = \dots \sin 2y(1 + \sin 2y)^{-1} \pm \dots \cos^2 2y(1 + \sin 2y)^{-2} \text{ or equivalent}$$

Condone invisible brackets for this mark

**A1\*** Correctly proceeds with no errors to the given answer including bracket errors.

**(c)**

**M1** Sets  $\frac{1 + \sin 2y}{2} = \frac{1}{4}$  and rearranges to  $\sin 2y = \dots$  Condone slips in the rearrangement for this mark.

**M1** Proceeds correctly to finding a value for  $y$  using their value for  $\sin 2y$

**A1**  $y = -\frac{\pi}{12}$  Ignore any sight of  $y = \frac{7\pi}{12}$

**M1** Substitutes their value for  $y$  into  $x = \frac{-\cos 2y}{1 + \sin 2y}$  and attempts to find a value for  $x$ . The expression may be unsimplified but the trigonometric functions must be evaluated and be exact.

**A1**  $x = -\sqrt{3}$  cao They do not have to write  $\left(-\sqrt{3}, -\frac{\pi}{12}\right)$  but withhold this mark if they have more than one pair of coordinates eg  $\left(\sqrt{3}, \frac{7\pi}{12}\right)$

## Specimen Paper WMA13/01

Question	Scheme	Marks
<b>10(a)</b>	$e^{2\alpha-3} - \frac{4}{3\alpha} = 0 \Rightarrow e^{2\alpha-3} = \dots \Rightarrow 2\alpha - 3 = \ln\left(\frac{4}{3\alpha}\right)$ $\alpha = \frac{1}{2}\left(\ln\left(\frac{4}{3\alpha}\right) + 3\right) \quad *$	M1 A1*
		<b>(2)</b>
<b>(b)</b>	$x_1 = \frac{1}{2}\left(\ln\left(\frac{4}{3 \times 2}\right) + 3\right) = \dots$ $x_1 = \text{awrt } 1.2973 \quad x_5 = \text{awrt } 1.4537$	M1 A1
		<b>(2)</b>
<b>(c)</b>	<p>Using <math>f(x) = e^{2x-3} - \frac{4}{3x}</math></p> $f(1.4555) = -0.0012199... < 0$ $f(1.4565) = 0.0012405... > 0$ <p>As <math>f(x)</math> is <u>continuous</u> on the required interval and there is a <u>change of sign</u> <math>\Rightarrow \underline{\alpha = 1.456}</math> (3 decimal places)</p>	M1 A1
		<b>(2)</b>
<b>(d)</b>	$\int_{-4}^{-2} \left( e^{2x-3} - \frac{4}{3x} \right) dx$ $\int \left( e^{2x-3} - \frac{4}{3x} \right) dx = \dots e^{2x-3} \pm \dots \ln x  \quad (+C)$ $\int \left( e^{2x-3} - \frac{4}{3x} \right) dx = \frac{1}{2} e^{2x-3} - \frac{4}{3} \ln x $ $= \left[ \frac{1}{2} e^{2x-3} - \frac{4}{3} \ln x  \right]_{-4}^{-2} = \left( \frac{1}{2} e^{-7} - \frac{4}{3} \ln -2  \right) - \left( \frac{1}{2} e^{-11} - \frac{4}{3} \ln -4  \right)$ $= \frac{1}{2} e^{-7} - \frac{1}{2} e^{-11} + \frac{4}{3} \ln 2$	M1 M1 A1 M1 A1
		<b>(5)</b>
<b>(11 marks)</b>		

## Specimen Paper WMA13/01

### Notes:

#### (a)

**M1** Sets  $y = 0$ , rearranges to  $e^{2\alpha-3} = \dots$  and takes lns of both sides. May still be in terms of  $x$

**A1\*** Achieves required form with no errors including brackets.

#### (b)

**M1** Substitutes  $x = 2$  into the given iteration formula and proceeds to finding  $x_1$ . May be implied by awrt 1.30

**A1**  $x_1 = \text{awrt } 1.2973$  and  $x_5 = \text{awrt } 1.4537$  only

#### (c)

**M1** Chooses a suitable interval eg (1.4555, 1.4565) and substitutes into the equation of the curve to find corresponding  $y$  values. Alternatively they may substitute into

$$\frac{1}{2} \left( \ln \left( \frac{4}{3x} \right) + 3 \right) - x = 0 \quad (\text{values are } 6.66 \times 10^{-4} \text{ and } -6.77 \times 10^{-4} \text{ to 3sf})$$

**A1** Correct values for their interval (may be rounded to 2 significant figures or truncated) Minimal conclusion stating that as the function is continuous on the required interval and there has been a change of sign this implies  $\alpha = 1.456$  to 3 decimal places.

#### (d)

**M1** A correct strategy to find the exact area of the shaded region. Sight of  $\int_{-4}^{-2} \left( e^{2x-3} - \frac{4}{3x} \right) dx$  is sufficient or it may be implied by an attempt to integrate and substitute in the limits either way round.

**M1** Attempts to integrate to a form  $\dots e^{2x-3} \pm \dots \ln|x|$  or  $\dots e^{2x-3} \pm \dots \ln|3x|$

**A1**  $\frac{1}{2} e^{2x-3} - \frac{4}{3} \ln|x|$  or exact equivalent. Note  $\ln|3x|$  is acceptable. Ignore (+C)

**M1** Substitutes  $-4$  and  $-2$  into their changed function and subtracts either way round

**A1**  $\frac{1}{2} e^{-7} - \frac{1}{2} e^{-11} + \frac{4}{3} \ln 2$  or exact simplified equivalent